Lecture 7

- Review
- Recombination and decoupling of photons (origin of CMB)
- Decoupling of neutrinos (Briefly)

Review

$$\frac{\partial n}{\partial t} - Hp\frac{\partial n}{\partial p} = I_{col.} \longrightarrow -\Gamma(n - n_{eq})$$

relaxation-time approx.

in case of $2 \to 2$

$$\Gamma = \tau^{-1} = \left(\left\langle \frac{\lambda}{v} \right\rangle\right)^{-1} = \langle \sigma N v \rangle$$

$$N \begin{cases} \simeq N_{eq}(T) & , \Gamma \gg H \\ \sim N_{eq}(T_*) \dfrac{a(T_*)^3}{a(T)^3} & , \Gamma \ll H \end{cases}$$

$\Gamma(T) \simeq H(T)$ is when freeze in or freeze out happens.

→ Explicit example with $e^+e^- \leftrightarrow \gamma\gamma$

$T_{out} \sim 10\ keV$

- Recombination and decoupling of photons (origin of CMB)

In the early universe atoms are highly ionized, there are a lot of free charges and the plazma is not transparent to photons. At some point the temperature falls below the binding energy of the Hydrogen atom, so it becomes favourable for them to form:

$$p + e \longrightarrow H + \gamma$$

Binding energy $I = 13.6\ eV \quad (1.5 \cdot 10^5\ K)$

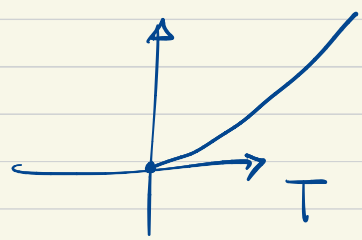
Note that by that time all anti-matter has anihilated.

After most charges are in the atoms photons propagate freely → Universe

becomes almost transparant. The time-slice when "the last" scattering occurs is called the last scattering surface.
CMB is a snapshot of the last scattering surface.

In more details, the densities follow Saha's equation, when in thermal eq.

$$\frac{n_e n_p}{n_H} = \left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-\frac{I}{T}}$$



Let's derive it:

$$n_e = g_e \left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-\frac{m_e-\mu_e}{T}}$$

(same for p, H)

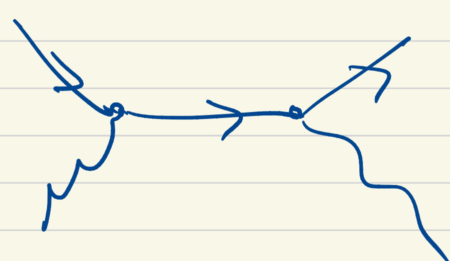
$g_e = g_p = 2 \qquad g_H = 4$

Chemical equilibrium: $\mu_p + \mu_e = \mu_H$ ($\mu_\gamma = 0$). Otherwise the reaction would change the distribution.

Now we can compute $\dfrac{n_p \cdot n_e}{n_H}$

Next we follow the following logic: We assume that plasma is in equilibrium and look for the temperature of decoupling of the Thompson scattering:

$e\gamma \to e\gamma$



This is a simplification. See Rubakov Gorbunov I, ch. 6 for a more precise treatment.

Nevertheless, let us proceed:

Neutrality of the U: $n_e = n_p$,
substituting into Saha gives

$$N_e = N_H^{1/2} \left(\frac{m_e T}{2\pi}\right)^{3/4} e^{-\frac{I}{2T}}$$

Now we need to determine the number of Baryons

$N_H + N_p = \text{const}$, as we will see soon $N_p \ll N_H$ at T of interest (most atoms recombined)

The ration of Baryons to Photons is a very important factor in cosmology. It is given by

$$\eta = \frac{N_B}{N_\gamma} = 6 \cdot 10^{-10}$$

(It is determined from CMB, BBN, etc., we will discuss later.)

Also fraction of He is $\sim 25\%$

We get

$$n_H \approx 0.75 \cdot \eta \frac{3(3)}{\pi^2} 2T^3$$

Remember that $\Gamma \simeq \langle \sigma_{\gamma e} n_e v \rangle$

We need cross-sections:

$$\sigma_{\gamma e} = \frac{8\pi\alpha^2}{3m_e^2}$$

This is why we can neglect

$$\sigma_{\gamma p} = \frac{8\pi\alpha^2}{3m_p^2}, \quad m_p \gg m_e$$

We get

$$\Gamma = \langle \sigma_{\gamma e} n_e v \rangle$$

Substituting in $\Gamma = H$ equation we get:

$$B \frac{T^{9/4}}{m_e^{5/4}} e^{-\frac{I}{2T}} \simeq \frac{T^2}{M_0}$$

[we will see shortly that we are in MD, so eq. for H is again an approx.]

$$B = \frac{8\pi\alpha^2}{3} \eta^{1/2} \left(\frac{2\zeta(3)}{\pi^2}\right)^{1/2} \left(\frac{1}{2\pi}\right)^{3/4}$$

Lets find the temperature:

$$x \equiv -\frac{I}{2T}$$

$$\frac{1}{x^{1/4}} e^{-x} = 1.2 \cdot 10^{-12}$$

$$x \simeq 27$$

$$T = \frac{I}{2 \cdot 27} = 0.25\,eV \quad (3000\,K)$$

temperature of CMB today: 2.73 K $\Rightarrow$

$$\Rightarrow z_{rec} = \frac{T_{dec}}{T_0} = 1100$$

(Matter-radiation equality: $z_{eq} = 3000$).

→ Recap the approximations...

Neutrinos

ν are very weakly interacting

$$\frac{\sigma_\nu}{\sigma_\gamma} \sim 10^{-18} \quad \text{at } E \sim 1\,\text{MeV}$$

$$\nu e \to \nu e$$

Earth and Sun are transparent for ν's

However, in the very early U. ν's were thermalized $\Rightarrow$ there is also a last scattering surface for ν's, same as for photons, just happens earlier.

$\rightsquigarrow$ At some point people thought that ν's can be dark matter, but now we know it is not the case due to more precise measurements.

Let us calculate the temperature and number density of Neutrinos.

Neutrino's cross-section:

$$\langle \sigma_w v \rangle \sim G_F^2 E^2 \quad , \; G_F = 10^{-5}\,\mathrm{GeV}^{-2}$$

$$\Gamma = \langle \sigma_w v n \rangle \sim G_F T^2 \cdot T^3$$

$$\Gamma = H \;\rightarrow\; T_*^{\nu} = (G_F^2 M_0)^{-1/3} \sim 2\,\mathrm{MeV}$$

Neutrinos are relativistic at decoupling, but are non-relativistic now*. Nevertheless they are described by the relativistic distribution with effective temperature

$$T_{\nu,0} = T_*^{\nu} \frac{a(T_*^{\nu})}{a(t_0)}$$

[this is what makes them a bad DM]

*It is still possible that one ν is massless

Importantly $T_{\nu,0} \neq T_{\gamma,0}$

this is because e^+e^- annihilation into γ's happens after ν decoupling. e^+ and e^- increase the temperature of photons but not of neutrinos.

→ To compute by how much we can use entropy conservation

$$g_*^{es} a^3 T^3 = \text{const}$$

$$g_*^{es}(T_*^{\nu}) = 2 + \frac{7}{8}(2+2) = \frac{11}{2}$$

$$g_*^{e\gamma}(T_0) = 2$$

$$\frac{T_{\gamma,0}}{T_{\nu,0}} = \left[\frac{g_*^{e\gamma}(T_{\nu,g})}{g_*^{e\gamma}(T_0)}\right]^{1/3} = \left(\frac{11}{4}\right)^{1/3} = 1.4$$

$$\Rightarrow T_{\nu,0} \simeq 2\,K$$

Cosmological bound on ν masses

$\rho_{\nu,0} = m_\nu N_{\nu,0}$, if $m_\nu > T_{\nu,0}$

$$\Omega_\nu = \frac{\rho_{\nu,0}}{\rho_c}$$

$$n_\nu = \overset{\text{fermions}}{\frac{3}{4}} \cdot \frac{g_\nu}{g_\gamma}\left(\frac{T_\nu}{T_\gamma}\right)^3 =$$

$= \frac{3}{22} n_\gamma$ (for each species)

$$\rho_{\nu,0} = \Sigma m_\nu \frac{3}{22} n_{\gamma,0}$$

For ν's not to be all DM we get

$\sum m_\nu \lesssim 10 \text{ eV}$

More detailed cosmological bounds give $\sum m_{\nu_i} \lesssim 0.2 \text{ eV}$ (stronger than direct exp.!)